



## OPTIMIZATION FOR COGNITIVE RADIO CHANNEL ALLOCATION STRATEGY WITH ACCESS PROBABILITY AND THRESHOLD\*

YUAN ZHAO, SHUNFU JIN<sup>†</sup> AND WUYI YUE

**Abstract:** In order to guarantee the Quality of Service (QoS) for the secondary users (SUs) in cognitive radio networks, in this paper, we propose a novel channel allocation strategy for the SUs by introducing a channel access probability and a channel access threshold, called the dynamic channel allocation strategy. In the networks, a newly arriving SU packet will join a common buffer with a channel access probability when the number of packets in the buffer is no less than the value of the channel access threshold. Based on the working principle of the proposed dynamic channel allocation strategy and the priority of the primary users (PUs) in cognitive radio networks, we first build a discrete-time pre-emptive priority queue to model the system operation. Then, by using a two-dimensional Markov chain, we analyze the queueing model in steady state, and derive some performance measures, such as the blocking ratio, the interruption ratio, the throughput and the average latency of the SUs. Moreover, by taking into account different performance measures for the system, we construct a net benefit function and present an iterative algorithm to optimize the channel access probability and the channel access threshold in the dynamic channel allocation strategy proposed in this paper. Additionally, we provide numerical results to show the influences of the channel access probability and the channel access threshold on the system performance measures, and also show the optimal results by operating the iterative algorithm proposed in this paper.

**Key words:** *cognitive radio, channel access probability, channel access threshold, pre-emptive priority queueing model, optimization, performance analysis and evaluation*

**Mathematics Subject Classification:** *68M20, 49K35*

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### 1 Introduction

Nowadays, a cognitive radio network has emerged as a novel technique used to alleviate the problem of under-utilization of the spectrum in the field of wireless communications [7, 13]. There are two types of users in cognitive radio networks, namely, primary users (PUs) and secondary users (SUs). The PUs have a pre-emptive priority to occupy the spectrum, and the SUs are allowed to access a licensed spectrum if the PUs are not occupying it. However, when a PU emerges, the SU occupying the spectrum has to quit the spectrum in order to avoid interference to the PU [9, 11].

As mentioned above, the data transmissions of the SUs are randomly interrupted by the PUs, and this will lead to a large number of SU packets staying in the buffer. On the other

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hand, in conventional cognitive radio networks, the SU packets would access to the system and wait in the buffer with probability 1. This will also lead to a longer latency for the SUs. Hence, the Quality of Service (QoS) of the SUs may be not ensured in cognitive radio networks, and it is necessary to improve the service level of the SUs [2, 14].

Recently, in order to guarantee the QoS of the SUs, some studies have been done to control the queueing behavior of the SUs in cognitive radio networks [8]. Konishi et al. considered a cognitive radio network with multi-channels, and each channel could be further divided into multiple sub-channels [5]. They assumed that the queueing behavior of the SUs could be controlled by applying a channel bonding scheme, with which an SU utilized multiple idle sub-channels. By using a continuous-time priority queueing model, they also analyzed the performance measures of the SUs, such as the blocking probability, the forced termination probability and the throughput. Moreover, they showed that there existed an optimal number of sub-channels to be bonded for an SU, while the throughput of the SUs achieved the highest value. Do et al. derived the average waiting time of packets with different priorities for the PUs and the SUs by employing a pre-emptive priority M/G/1 queueing model [4]. They found an optimal way to allocate how many packets should enter the system in cognitive radio networks. Turhan et al. studied a cognitive radio system with multi-channels [12]. They proposed an admission control policy for the SUs with threshold. When the total number of users in the system was less than the threshold, a newly arriving SU was accepted. Otherwise, the SU was rejected. Moreover, they modeled the system as an M/M/C/C queueing and computed the optimal admission control of the SUs which maximized the profit of the system. Li et al. assumed that there was a queue threshold set for the SUs in a cognitive radio network [6]. When the current queue length reached the threshold, the SUs left. With Nash equilibrium, they tried to give the individually and socially optimal thresholds of the SUs.

However, in the channel allocation strategies mentioned above, there was no channel access threshold at all, or when the number of SU packets reached the channel access threshold, the new arrival SU packet would not access the channel with probability 1. For these cases, both the blocking ratio and the average latency of the SUs were higher, or the throughput of the SUs was lower.

On the other hand, most of the related studies on performance evaluation of the cognitive radio networks with queueing theory were considered in the continuous-time field. However, communication systems are more often digital in modern times. It would be more accurate and efficient to use discrete-time queues than their continuous counterparts to model the system operation when analyzing and designing digital transmitting systems [1].

Additionally, with the aim to simplify the analysis, in most of the existing performance researches on the channel allocation strategies in cognitive radio networks, the buffer for the SUs was assumed to be infinite. Since a cognitive radio network may have greater number of SU packet arrivals, and the buffer capacity of the SUs is more important in the analysis of cognitive radio networks. Consequently, it is more reasonable to assume a finite buffer in system analysis of cognitive radio networks.

In this paper, in order to guarantee the QoS for the SUs, we propose a novel channel allocation strategy by introducing a channel access probability and a channel access threshold for the SUs. We call this channel allocation strategy the dynamic channel allocation strategy with channel access probability and channel access threshold. When the number of SU packets in the buffer is no less than the value of the channel access threshold, a newly arriving SU packet will choose to join the system with a channel access probability. According to the working principle of the dynamic channel allocation strategy proposed in this paper, and by considering the digital nature in the cognitive radio networks, we can build a pre-emptive

priority discrete-time queue with finite buffer capacity to model the system operation. For the purpose of obtaining the steady-state distribution of this queueing model, we construct a two-dimensional Markov chain and give the transition probability matrix of this Markov chain. Accordingly, we derive the performance measures of the system, such as the blocking ratio, the interruption ratio, the throughput and the average latency of the SUs. This paper is an extended version of our conference paper presented in [16]. This paper is substantially different from the conference paper of [16] to give essential and useful analyses showing the influences of both the channel access probability and the channel access threshold on the system performances not only by numerical results, but by performance analysis. Furthermore, by combining different performance measures, we present a net benefit function and give an iterative algorithm to optimize the channel access probability and the channel access threshold.

The remainder of this paper is organized as follows. A dynamic channel allocation strategy is proposed, then the system model is built in Section 2. The performance analysis is given in Section 3. In Section 4, the formulas for the performance measures, such as the blocking ratio, the interruption ratio, the throughput and the average latency of the SUs are obtained. The optimizations of the channel access probability and the channel access threshold are presented in Section 5. Numerical results for the system performance and system optimization are provided in Section 6. Finally, conclusions are drawn in Section 7.

## **2 A Novel Channel Allocation Strategy and System Model**

### **2.1 The Dynamic Channel Allocation Strategy with Channel Access Probability and Channel Access Threshold for the SUs**

In the centralized cognitive radio network with downlink cellular scenario, there are two types of users, namely, primary users (PUs) and secondary users (SUs). The PUs have a pre-emptive priority to occupy the channel, and the SUs can just make opportunistic use of the channel. Moreover, an interrupted SU packet is assumed to have a higher priority to a newly arriving SU packet.

In order to reduce the blocking ratio of the SUs, a buffer with finite capacity  $K$  ( $K > 0$ ) is deployed for the SU packets. On the other hand, in order to best satisfy the latency need of the PUs, no buffer is prepared for the PU packets.

Moreover, for the purpose of reducing the average latency of the SU packets, we set a channel allocation strategy with a channel access probability  $\alpha$  and a channel access threshold  $H$  for the SUs, where  $0 < \alpha \leq 1$  and  $0 < H \leq K$ . We call this channel allocation strategy the dynamic channel allocation strategy with channel access probability and channel access threshold.

There is a central controller which can broadcast different messages about the system, including the number of SU packets in the buffer, to the network users [6]. When a packet of an SU arrives at the system, the central controller can send a message to that SU packet about the number of SU packets already in the buffer. When the number of SU packets in the SU buffer is less than  $H$ , this SU packet will join the system with probability 1. On the other hand, if the number of SU packets in the SU buffer is equal to or more than  $H$ , this SU packet will choose to join the system with the channel access probability  $\alpha$ , and leave the system with probability  $(1 - \alpha)$ .

The working principle of the dynamic channel allocation strategy proposed in this paper is illustrated in Fig. 1.

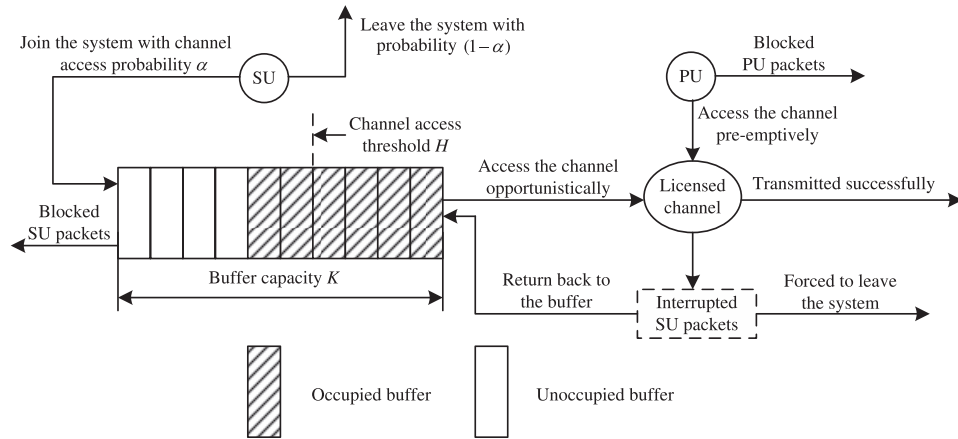


Figure 1: The working principle of the dynamic channel allocation strategy.

From Fig. 1, we can describe the actions of the PU packets and the SU packets in the network as follows:

- (1) When an SU packet chooses to join the system, if the channel is being occupied by another packet, this SU packet will queue in the buffer of the SUs. If the buffer of the SUs is full, this SU packet will be blocked by the system.
- (2) Since the PUs have a pre-emptive priority to occupy the channel, when a PU packet arrives at the system, there are three possibilities. (a) If the channel is idle, this PU packet will certainly occupy the channel directly. (b) If the channel is being occupied by another PU packet, the newly arriving PU packet will be blocked. (c) If the channel is being occupied by an SU packet, this PU packet will interrupt the transmission of this SU packet and occupy the channel immediately.
- (3) When the transmission of an SU packet is interrupted by a PU packet, if the buffer of the SUs is full, the interrupted SU packet will be forced to leave the system. Otherwise, the interrupted SU packet will return to the buffer of the SUs and queue at the head. Note that the interrupted SU packets have a higher priority than the newly arriving SU packets. Given that there is only one vacancy in the buffer of the SUs, if a new arrival of an SU packet and an interruption of an SU packet occur simultaneously, the interrupted SU packet will join the system, and the newly arriving SU packet will be blocked by the system.

## 2.2 System Model

We consider a cognitive radio network with single-channel. The channel is regarded as a server, and the PU packets and the SU packets are seen as two types of customers. The SU buffer can be seen as a waiting room. According to the working principle of the dynamic channel allocation strategy with channel access probability and channel access threshold, we can build a pre-emptive priority queueing model with two types of customers in this paper. Additionally, in the following paper, in order to avoid ambiguity, we will equalize the terms “customer” and “packet”.

A slotted timing structure is considered in this system model for digitized communication, in which the time axis is divided into slots with equal size, and the slot boundaries are numbered by  $t = n$  ( $n = 1, 2, \dots$ ). It is assumed that all events occur at the slot boundaries, and of course different events can occur at the same epoch. So it is important to clearly specify the occurring sequence of different events. The packets are supposed to arrive immediately after the beginning instant of a slot, and depart just prior to the end of a slot. It means that, the potential arrival of a packet can occur in the interval  $(n, n^+)$ , and the potential departure of a packet can occur in the interval  $(n^-, n)$ . Namely, an early arrival queueing model is employed.

Specially, in order to avoid ambiguity, we assume the central controller can inform a newly arriving SU packet the number of SU packets already in the buffer at  $t = n^-$ , and the newly arriving SU packet can decide whether or not to join the system based on the information received from the central controller. In our model, we call an epoch at which the SU packets make the joining decisions as a decision epoch.

Taking  $t = n$  and  $t = n + 1$  as examples, various time epochs at which different events occur are depicted in Fig. 2.

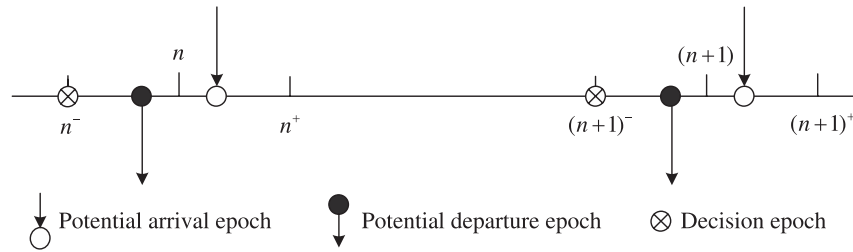


Figure 2: Various time epochs in the system model.

The arriving intervals and transmission times of the PU packets and the SU packets are supposed to be independent, identically distributed (i.i.d) random variables. Comply with the slotted structure, the arriving intervals and transmission times of the PU packets are assumed to follow geometrical distributions with parameters  $\lambda_1$  ( $0 < \lambda_1 < 1, \bar{\lambda}_1 = 1 - \lambda_1$ ) and  $\mu_1$  ( $0 < \mu_1 < 1, \bar{\mu}_1 = 1 - \mu_1$ ), respectively. On the other hand, the arriving intervals and transmission times of the SU packets are assumed to follow geometrical distributions with parameters  $\lambda_2$  ( $0 < \lambda_2 < 1, \bar{\lambda}_2 = 1 - \lambda_2$ ) and  $\mu_2$  ( $0 < \mu_2 < 1, \bar{\mu}_2 = 1 - \mu_2$ ), respectively.

Let  $L_n = i$ ,  $i \in \{0, 1, 2, \dots, K + 1\}$  be the total number of packets in the system at  $t = n^+$ , and  $L_n^{(1)} = j$ ,  $j \in \{0, 1\}$  be the number of PU packets in the system at  $t = n^+$ , where  $K$  is the capacity of the SU buffer.  $\{L_n, L_n^{(1)}\}$  constitutes a two-dimensional Markov chain with the state space  $\Omega$  given as follows:

$$\Omega = (0, 0) \cup \{(i, j) : 1 \leq i \leq K + 1, j = 0, 1\}.$$

### 3 Performance Analysis

#### 3.1 State Transition Probability Matrix

We define the system level as the total number of packets in the system at  $t = n^+$ , where the system level ranges from 0 to  $(K + 1)$ . According to the state transition of the system levels,



where  $\mathbf{P}_{1,0}$  is a column vector with two elements, and  $\mathbf{T}$  is the transpose operator of the matrix.

(b) Given that the packet in the system at  $t = n^+$  is an SU packet, there will be an SU packet and no PU packet in the system at  $t = (n+1)^+$  with probability  $\bar{\lambda}_1(\bar{\lambda}_2\bar{\mu}_2 + \lambda_2\mu_2)$ ; there will be no SU packet and a PU packet in the system at  $t = (n+1)^+$  with probability  $\lambda_1\bar{\lambda}_2\mu_2$ . Given that the packet in the system at  $t = n^+$  is a PU packet, there will be an SU packet and no PU packet in the system at  $t = (n+1)^+$  with probability  $\bar{\lambda}_1\lambda_2\mu_1$ ; there will be no SU packet and a PU packet in the system at  $t = (n+1)^+$  with probability  $\bar{\lambda}_2(\lambda_1\mu_1 + \bar{\mu}_1)$ . For these cases, there will be one packet in the system at  $t = (n+1)^+$ , namely,  $v = 1$ .  $\mathbf{P}_{1,1}$  can be given by

$$\mathbf{P}_{1,1} = \begin{pmatrix} \bar{\lambda}_1(\bar{\lambda}_2\bar{\mu}_2 + \lambda_2\mu_2) & \lambda_1\bar{\lambda}_2\mu_2 \\ \bar{\lambda}_1\lambda_2\mu_1 & \bar{\lambda}_2(\lambda_1\mu_1 + \bar{\mu}_1) \end{pmatrix} \quad (3.5)$$

where  $\mathbf{P}_{1,1}$  is a  $2 \times 2$  square matrix.

(c) Given that the packet in the system at  $t = n^+$  is an SU packet, there will be two SU packets and no PU packet in the system at  $t = (n+1)^+$  with probability  $\bar{\lambda}_1\lambda_2\bar{\mu}_2$ ; there will be an SU packet and a PU packet in the system at  $t = (n+1)^+$  with probability  $\lambda_1(\bar{\lambda}_2\bar{\mu}_2 + \lambda_2\mu_2)$ . Given that the packet in the system at  $t = n^+$  is a PU packet, there will be two SU packets and no PU packet in the system at  $t = (n+1)^+$  with probability 0; there will be an SU packet and a PU packet in the system at  $t = (n+1)^+$  with probability  $\lambda_2(\lambda_1\mu_1 + \bar{\mu}_1)$ . For these cases, there will be two packets in the system at  $t = (n+1)^+$ , namely,  $v = 2$ .  $\mathbf{P}_{1,2}$  can be given by

$$\mathbf{P}_{1,2} = \begin{pmatrix} \bar{\lambda}_1\lambda_2\bar{\mu}_2 & \lambda_1(\bar{\lambda}_2\bar{\mu}_2 + \lambda_2\mu_2) \\ 0 & \lambda_2(\lambda_1\mu_1 + \bar{\mu}_1) \end{pmatrix} \quad (3.6)$$

where  $\mathbf{P}_{1,2}$  is a  $2 \times 2$  square matrix.

(d) Given that the packet in the system at  $t = n^+$  is an SU packet, there will be three SU packets and no PU packet in the system at  $t = (n+1)^+$  with probability 0; there will be two SU packets and a PU packet in the system at  $t = (n+1)^+$  with probability  $\lambda_1\lambda_2\bar{\mu}_2$ . Given that the packet in the system at  $t = n^+$  is a PU packet, there will be three SU packets and no PU packet in the system at  $t = (n+1)^+$  with probability 0; there will be two SU packets and a PU packet in the system at  $t = (n+1)^+$  with probability 0. For these cases, there will be three packets in the system at  $t = (n+1)^+$ , namely,  $v = 3$ .  $\mathbf{P}_{1,3}$  can be given by

$$\mathbf{P}_{1,3} = \begin{pmatrix} 0 & \lambda_1\lambda_2\bar{\mu}_2 \\ 0 & 0 \end{pmatrix} \quad (3.7)$$

where  $\mathbf{P}_{1,3}$  is a  $2 \times 2$  square matrix.

- (3) For the system level  $2 \leq u \leq \min\{H, K-1\}$ , namely, the channel is occupied and the number of packets in the buffer is less than the channel access threshold at  $t = n^+$ , there are four  $2 \times 2$  non-zero sub-blocks for  $v = u-1, u, u+1, u+2$  in  $\mathbf{P}$  as follows:

$$\mathbf{P}_{u,v} = \begin{cases} (\mathbf{P}_{1,0}, \mathbf{0}_{2 \times 1}), & v = u-1 \\ \mathbf{P}_{1,1}, & v = u \\ \mathbf{P}_{1,2}, & v = u+1 \\ \mathbf{P}_{1,3}, & v = u+2 \end{cases} \quad (3.8)$$

where  $\mathbf{0}_{2 \times 1}$  is a zero's column vector with two elements.  $\mathbf{P}_{1,0}$ ,  $\mathbf{P}_{1,1}$ ,  $\mathbf{P}_{1,2}$  and  $\mathbf{P}_{1,3}$  can be obtained by referencing Eqs. (3.4)-(3.7).

- (4) For the system level  $H + 1 \leq u \leq K - 1$ , namely, the channel is occupied and the number of packets in the buffer is equal to or larger than the channel access threshold at  $t = n^+$ , the arrival rate of the SU packets for this case will be  $\lambda_2\alpha$ . Similar to the matrix structure shown in Eqs. (3.4)-(3.7), there are also four non-zero sub-blocks for  $v = u - 1, u, u + 1, u + 2$  in  $\mathbf{P}$ .

(a) When  $v = u - 1$ ,  $\mathbf{P}_{u,u-1}$  is a  $2 \times 2$  square matrix given as follows:

$$\mathbf{P}_{u,u-1} = \begin{pmatrix} \bar{\lambda}_1(1 - \lambda_2\alpha)\mu_2 & 0 \\ \bar{\lambda}_1(1 - \lambda_2\alpha)\mu_1 & 0 \end{pmatrix}. \quad (3.9)$$

(b) When  $v = u$ ,  $\mathbf{P}_{u,u}$  is a  $2 \times 2$  square matrix given as follows:

$$\mathbf{P}_{u,u} = \begin{pmatrix} \bar{\lambda}_1((1 - \lambda_2\alpha)\bar{\mu}_2 + \lambda_2\alpha\mu_2) & \lambda_1(1 - \lambda_2\alpha)\mu_2 \\ \bar{\lambda}_1\lambda_2\alpha\mu_1 & (1 - \lambda_2\alpha)(\lambda_1\mu_1 + \bar{\mu}_1) \end{pmatrix}. \quad (3.10)$$

(c) When  $v = u + 1$ ,  $\mathbf{P}_{u,u+1}$  is a  $2 \times 2$  square matrix given as follows:

$$\mathbf{P}_{u,u+1} = \begin{pmatrix} \bar{\lambda}_1\lambda_2\alpha\bar{\mu}_2 & \lambda_1((1 - \lambda_2\alpha)\bar{\mu}_2 + \lambda_2\alpha\mu_2) \\ 0 & \lambda_2\alpha(\lambda_1\mu_1 + \bar{\mu}_1) \end{pmatrix}. \quad (3.11)$$

(d) When  $v = u + 2$ ,  $\mathbf{P}_{u,u+2}$  is a  $2 \times 2$  square matrix given as follows:

$$\mathbf{P}_{u,u+2} = \begin{pmatrix} 0 & \lambda_1\lambda_2\alpha\bar{\mu}_2 \\ 0 & 0 \end{pmatrix}. \quad (3.12)$$

- (5) For the system level  $u = K$ , namely, there is only one vacancy in the buffer at  $t = n^+$ , there are three non-zero sub-blocks for  $v = K - 1, K, K + 1$  in  $\mathbf{P}$ . With these three non-zero sub-blocks, we discuss the form of  $\mathbf{P}_{K,v}$  in two cases.

For the case of  $0 < H < K$ , that is to say, the channel access threshold is smaller than the buffer capacity,  $\mathbf{P}_{K,v}$  can be given as follows:

$$\mathbf{P}_{K,v} = \begin{cases} \mathbf{P}_{H+1,H}, & v = K - 1 \\ \mathbf{P}_{H+1,H+1}, & v = K \\ \mathbf{P}_{H+1,H+2} + \mathbf{P}_{H+1,H+3}, & v = K + 1 \end{cases} \quad (3.13)$$

where  $\mathbf{P}_{H+1,H}$ ,  $\mathbf{P}_{H+1,H+1}$ ,  $\mathbf{P}_{H+1,H+2}$  and  $\mathbf{P}_{H+1,H+3}$  can be obtained from Eqs. (3.9)-(3.12), respectively.

For the case of  $H = K$ , that is to say, the channel access threshold is equal to the buffer capacity,  $\mathbf{P}_{K,v}$  can be given in two possibilities.

When  $K > 1$ ,  $\mathbf{P}_{K,v}$  can be given as follows:

$$\mathbf{P}_{K,v} = \begin{cases} (\mathbf{P}_{1,0}, \mathbf{0}_{2 \times 1}), & v = K - 1 \\ \mathbf{P}_{1,1}, & v = K \\ \mathbf{P}_{1,2} + \mathbf{P}_{1,3}, & v = K + 1. \end{cases} \quad (3.14)$$



When  $K = 1$ ,  $\mathbf{P}_{K,v}$  can be given as follows:

$$\mathbf{P}_{K,v} = \begin{cases} \mathbf{P}_{1,0}, & v = K - 1 \\ \mathbf{P}_{1,1}, & v = K \\ \mathbf{P}_{1,2} + \mathbf{P}_{1,3}, & v = K + 1. \end{cases} \quad (3.15)$$

In Eqs. (3.14) and (3.15),  $\mathbf{P}_{1,0}$ ,  $\mathbf{P}_{1,1}$ ,  $\mathbf{P}_{1,2}$  and  $\mathbf{P}_{1,3}$  can be obtained from Eqs. (3.4)-(3.7), respectively.

- (6) For the system level  $u = K + 1$ , namely, there is no vacancy in the buffer at  $t = n^+$ , there are two non-zero sub-blocks for  $v = K, K + 1$  in  $\mathbf{P}$  as follows:

$$\mathbf{P}_{K+1,v} = \begin{cases} \mathbf{P}_{H+1,H}, & v = K \\ \mathbf{P}_{H+1,H+1} + \mathbf{P}_{H+1,H+2} + \mathbf{P}_{H+1,H+3}, & v = K + 1 \end{cases} \quad (3.16)$$

where  $\mathbf{P}_{H+1,H}$ ,  $\mathbf{P}_{H+1,H+1}$ ,  $\mathbf{P}_{H+1,H+2}$  and  $\mathbf{P}_{H+1,H+3}$  can be obtained from Eqs. (3.9)-(3.12), respectively.

### 3.2 Steady-State Distribution

The structure of the transition probability matrix  $\mathbf{P}$  indicates that the two-dimensional Markov chain  $\{L_n, L_n^{(1)}\}$  is non-periodic, irreducible and positive recurrent. Let  $\pi_{i,j}$  denote the steady-state distribution of this two-dimensional Markov chain.  $\pi_{i,j}$  can be given as follows:

$$\pi_{i,j} = \lim_{n \rightarrow \infty} P \{L_n = i, L_n^{(1)} = j\}. \quad (3.17)$$

Let  $\mathbf{\Pi}_i$  be the steady-state probability vector for the system being at level  $i$ .  $\mathbf{\Pi}_i$  can be given as follows:

$$\mathbf{\Pi}_i = \begin{cases} \pi_{0,0}, & i = 0 \\ (\pi_{i,0}, \pi_{i,1}), & 1 \leq i \leq K + 1. \end{cases} \quad (3.18)$$

$\mathbf{\Pi}_i$  can be calculated by solving the following equilibrium equation

$$(\mathbf{\Pi}_0, \mathbf{\Pi}_1, \dots, \mathbf{\Pi}_K, \mathbf{\Pi}_{K+1})\mathbf{P} = (\mathbf{\Pi}_0, \mathbf{\Pi}_1, \dots, \mathbf{\Pi}_K, \mathbf{\Pi}_{K+1}) \quad (3.19)$$

with the normalization condition

$$(\mathbf{\Pi}_0, \mathbf{\Pi}_1, \dots, \mathbf{\Pi}_K, \mathbf{\Pi}_{K+1})\mathbf{e} = 1 \quad (3.20)$$

where  $\mathbf{e}$  is a column vector with  $2 \times (K + 1) + 1$  elements, all of which equal 1.

Substituting Eq. (3.18) to Eqs. (3.19) and (3.20), we can get a multivariate system of linear equations with  $2 \times (K + 1) + 1$  unknowns. By solving the linear equations with a simple Gauss-Seidel method [15], we can obtain the steady-state distribution  $\pi_{i,j}$  defined in Eq. (3.17).

## 4 Performance Measures

We define the average access ratio  $\beta$  of the SUs as the average number of SU packets which choose to join the system per slot. According to the dynamic channel allocation strategy

with channel access probability and channel access threshold proposed in this paper,  $\beta$  can be given as follows:

$$\beta = \lambda_2 \pi_{0,0} + \lambda_2 \sum_{i=1}^H (\pi_{i,0} + \pi_{i,1}) + \lambda_2 \alpha \sum_{i=H+1}^{K+1} (\pi_{i,0} + \pi_{i,1}) \quad (4.1)$$

where  $\alpha$  is the channel access probability, and  $H$  is the channel access threshold.

We define the blocking ratio  $P_B$  of the SUs as the average number of newly arriving SU packets that are blocked by the system per slot. A newly arriving SU packet will be blocked by the system when this SU packet chooses to join the system, but finds the total number of packets in the system is already  $(K + 1)$ . That is to say, the buffer of the SUs is full, and the channel is occupied. Therefore, the blocking ratio  $P_B$  of the SUs can be given as follows:

$$P_B = \begin{cases} \lambda_2 \alpha ((\bar{\mu}_2 + \mu_2 \lambda_1) \pi_{K+1,0} + (\bar{\mu}_1 + \mu_1 \lambda_1) \pi_{K+1,1} + \lambda_1 \bar{\mu}_2 \pi_{K,0}), & 0 < H < K \\ \lambda_2 \alpha ((\bar{\mu}_2 + \mu_2 \lambda_1) \pi_{K+1,0} + (\bar{\mu}_1 + \mu_1 \lambda_1) \pi_{K+1,1}) + \lambda_1 \lambda_2 \bar{\mu}_2 \pi_{K,0}, & H = K. \end{cases} \quad (4.2)$$

We define the interruption ratio  $P_I$  of the SUs as the average number of SU packets that are interrupted by a PU packet per slot. An SU packet being transmitted will be interrupted when a PU packet arrives at the system during the transmission time of this SU packet. Therefore, the interruption ratio  $P_I$  of the SUs can be given as follows:

$$P_I = \lambda_1 \bar{\mu}_2 \sum_{i=1}^{K+1} \pi_{i,0}. \quad (4.3)$$

We define the interrupted blocking ratio  $P_{IB}$  of the SUs as the average number of SU packets that are interrupted by a new arrival of a PU packet and forced to leave the system due to lack of buffer capacity at the interruption. Therefore, the interrupted blocking ratio  $P_{IB}$  of the SUs can be given as follows:

$$P_{IB} = \lambda_1 \bar{\mu}_2 \pi_{K+1,0}. \quad (4.4)$$

We define the throughput  $S$  of the SUs as the average number of SU packets transmitted successfully per slot. An SU packet can be transmitted successfully if and only if this SU packet chooses to join the system, and this SU packet is not blocked by the system, nor forced to leave the system before the transmission is completely finished. Therefore, the throughput  $S$  of the SUs can be given as follows:

$$S = \beta - P_B - P_{IB}. \quad (4.5)$$

We define the latency of an SU packet as the time period from the instant that an SU packet arrives at the system and chooses to access the channel to the instant that this SU packet is completely transmitted.

Letting  $L_n^{(2)}$  be the number of SU packets in the system at  $t = n^+$  and  $L^{(2)}$  be the steady-state distribution of  $L_n^{(2)}$ , then  $L^{(2)}$  can be given as follows:

$$L^{(2)} = \lim_{n \rightarrow \infty} L_n^{(2)}. \quad (4.6)$$

Accordingly, we can get the average number of SU packets in steady state as follows:

$$E[L^{(2)}] = \sum_{j=0}^{K+1} j P\{L^{(2)} = j\} = \sum_{j=1}^K j (\pi_{j,0} + \pi_{j+1,1}) + (K+1) \pi_{K+1,0}. \quad (4.7)$$

By referencing Little's law [10], the average latency  $\delta$  of the SUs can be given as follows:

$$\delta = \frac{E[L^{(2)}]}{S} = \frac{\sum_{j=1}^K j(\pi_{j,0} + \pi_{j+1,1}) + (K+1)\pi_{K+1,0}}{S} \quad (4.8)$$

where  $S$  can be obtained from Eq. (4.5).

## 5 System Optimization

In this section, we analyze how to get a net benefit function to optimize the channel access probability and the channel access threshold.

In practice, as the channel access probability and the channel access threshold increase, the throughput of the SUs will also increase. This is a "good thing". On the other hand, as the channel access probability and the channel access threshold increase, the blocking ratio, the interruption ratio and the average latency of the SUs will increase. These are "bad things". Therefore, it is necessary to balance different performance measures. In order to optimize the channel access probability and the channel access threshold, we construct a net benefit function  $F(H, \alpha)$  as follows:

$$F(H, \alpha) = C_1 S - C_2 \delta - C_3 P_B - C_4 P_I \quad (5.1)$$

where  $C_1$  is the reward due to the throughput of the SUs,  $C_2$ ,  $C_3$  and  $C_4$  are the costs due to the average latency, the blocking ratio and the interruption ratio of the SUs on the net benefit, respectively.

In practice, for the networks with higher throughput sensitive,  $C_1$  should be set larger. On the other hand, for the networks with lower tolerance to the average latency, the blocking ratio and the interruption ratio,  $C_2$ ,  $C_3$  and  $C_4$  should be set larger.

From Eq. (5.1), the optimal value  $(H^*, \alpha^*)$  can be given as follows:

$$(H^*, \alpha^*) = \arg \max\{F(H, \alpha)\} \quad (5.2)$$

where "arg max" stands for the argument of the maximum [3].

Since the explicit solution for the steady-state distribution of the system can not be derived for the system model in this paper, so it is difficult to present the results for the optimal value  $(H^*, \alpha^*)$  in a close form.

Due to the fact that  $H$  is a discrete quantity, and  $\alpha$  is a continuous quantity, we attempt to firstly obtain an approximate solution  $\alpha^*$  with maximum  $F(H, \alpha^*)$  for various values of  $H$ .

As in the following optimization, we will propose an iteration algorithm based on an unconstrained minimization technique. Considering the constraint of  $\alpha$ , i.e.,  $0 < \alpha \leq 1$ , we construct a penalty term  $C(\alpha)$  as follows:

$$C(\alpha) = \frac{1}{\alpha - \alpha_{min}} + \frac{1}{\alpha_{max} - \alpha} \quad (5.3)$$

where  $\alpha_{min}$  and  $\alpha_{max}$  are the lower limit and the upper limit of  $\alpha$ , respectively. The role of the penalty term is transferring constrained condition into unconstrained condition in the optimization problem.

Then, we build a penalty function as follows:

$$B(H, \alpha) = -F(H, \alpha) + rC(\alpha) \quad (5.4)$$

where  $r > 0$  is the penalty factor.

Given that the product of  $r$  and  $C(\alpha)$  is small enough, with the value  $(H, \alpha)$  for minimizing  $B(H, \alpha)$ , we can get the maximal  $F(H, \alpha)$ . In order to obtain the optimal value  $(H^*, \alpha^*)$  with  $B(H, \alpha)$  defined in Eq. (5.2), by referencing the steepest descent method [3], we present an iterative algorithm in Table 1.

Table 1: Iteration algorithm to find the optimal solution of Eq. (5.2).

---

**Algorithm:** Find the optimal solution of Eq. (5.2).

---

**Step 1:** For each value of  $H$ , set the initial value for  $\alpha_m$  with  $m = 0$ .

**Step 2:** Compute the new value  $\alpha_{m+1} = \alpha_m - \phi \frac{\partial B(H, \alpha)}{\partial \alpha} \Big|_{\alpha=\alpha_m}$ ,  
where  $\phi$  is the step size.

**Step 3:** Set  $m = m + 1$  and repeat **Step 2**, if  $|B(H, \alpha_{m+1}) - B(H, \alpha_m)| > \epsilon$  or  $|\alpha_{m+1} - \alpha_m| > \epsilon$ , where  $\epsilon$  is tolerance;  
Otherwise, go to **Step 4**.

**Step 4:** Compute  $rC(\alpha_{m+1})$ .  
If  $rC(\alpha_{m+1}) > \epsilon$ , repeat **Steps 1-3** by setting  $\alpha_0 = \alpha_{m+1}$ ,  $r = rD$ ,  
where  $D$  is the decline coefficient of the penalty factor  $r$ ;  
Otherwise, go to **Step 5**.

**Step 5:** Obtain  $\alpha^* = \alpha_{m+1}$  with the minimum  $B(H, \alpha^*)$ ,  
and then compute the corresponding  $F(H, \alpha^*)$ .

**Step 6:** Find the optimal solution  $(H^*, \alpha^*)$  from all the values of  $F(H, \alpha^*)$   
obtained in **Step 5**.

---

The partial derivative of  $B(H, \alpha)$  for  $\alpha$  in this algorithm can be approximated numerically as follows:

$$\frac{\partial B(H, \alpha)}{\partial \alpha} \approx \frac{B(H, \alpha + \Delta) - B(H, \alpha)}{\Delta}$$

where  $\Delta$  is an arbitrary small number (for example,  $\Delta = 10^{-6}$ ).

The convergence for the algorithm mentioned above can reference the proof for the convergence of steepest descent method given in [3].

## 6 Numerical Results

In this section, we provide numerical results to show the change trends of the different performance measures and also the optimal design for the channel access probability and the channel access threshold.

In the numerical results, the arrival rate of the PU packets is set as  $\lambda_1 = 0.16$ ; the arrival rate of the SU packets is set as  $\lambda_2 = 0.12$ ; the transmission rate of the SU packets is set as  $\mu_2 = 0.15$ ; the buffer capacity of the SUs is set as  $K = 10$ . We note here that the above parameters setting for  $\lambda_1$ ,  $\lambda_2$ ,  $\mu_2$  and  $K$  is just an example, and another choice for the parameters setting does not change our conclusions for the numerical results.

Figures 3-5 show how the blocking ratio  $P_B$ , the interruption ratio  $P_I$  and the average latency  $\delta$  of the SUs change with the channel access probability  $\alpha$  for the different channel access thresholds  $H$ , respectively. Additionally, the data set of the channel access probability  $\alpha$  is assumed to be  $\{0.1, 0.2, \dots, 0.9, 1.0\}$ .

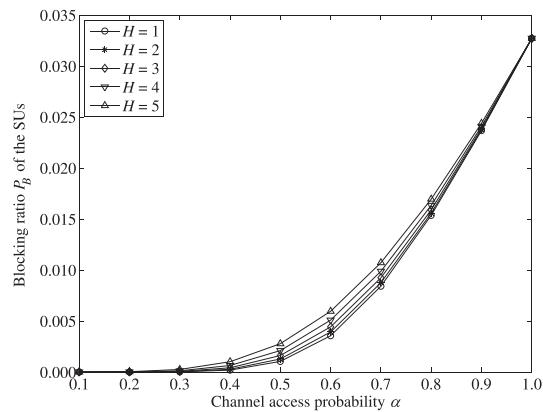


Figure 3: Blocking ratio  $P_B$  of the SUs vs. channel access probability  $\alpha$  ( $\mu_1 = 0.21$ ).

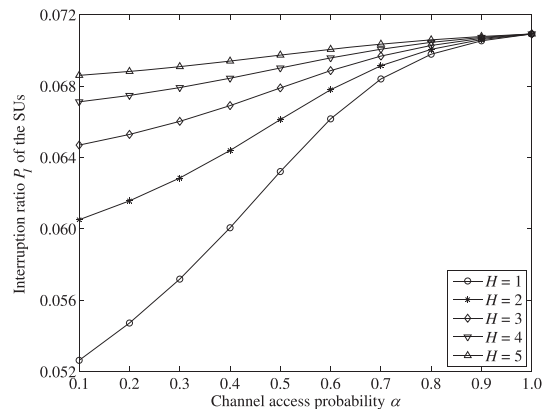


Figure 4: Interruption ratio  $P_I$  of the SUs vs. channel access probability  $\alpha$  ( $\mu_1 = 0.21$ ).

In Figs. 3-5, we notice that for the same channel access threshold  $H$ , as the channel access probability  $\alpha$  increases, the blocking ratio  $P_B$ , the interruption ratio  $P_I$  and the average latency  $\delta$  of the SUs will increase. The reason is that for the same channel access threshold, when the number of SU packets in the system exceeds the channel access threshold, the larger the channel access probability is, the more likely it is that a newly arriving SU packet will choose to access the channel. When a newly arriving SU packet chooses to access the channel, if the system buffer is already full, this SU packet will be blocked, so the blocking ratio of the SUs will increase. On the other hand, if there is at least one vacancy in the buffer, this SU packet will join the system, then the number of SU packets in the system will increase, and then the average latency of the SUs will be greater. After this newly arriving SU packet joins the system buffer, if the transmission of this SU packet is interrupted, the interruption ratio of the SUs will be greater.

We also observe that for the same channel access probability  $\alpha$ , along with an increase in the channel access threshold  $H$ , the blocking ratio  $P_B$ , the interruption ratio  $P_I$  and the average latency  $\delta$  of the SUs will increase. This is because the higher the channel access threshold is, the less likely it is that the system will reach the threshold. Note that a newly arriving SU packet will definitively join the system if the number of SU packets in the

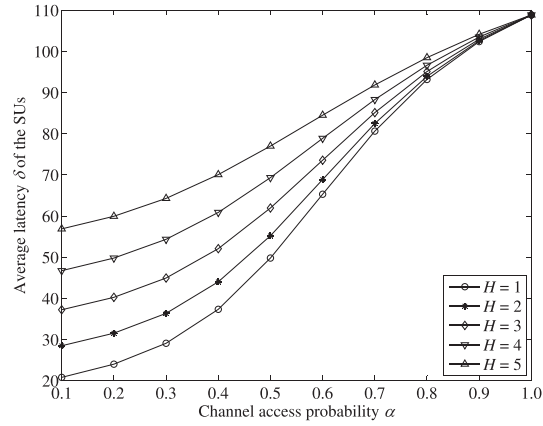


Figure 5: Average latency  $\delta$  of the SUs vs. channel access probability  $\alpha$  ( $\mu_1 = 0.21$ ).

buffer is less than the channel access threshold, and join the system with the channel access probability  $\alpha$  if the number of packets in the buffer is equal to or larger than the channel access threshold. It is obvious that when the channel access threshold is higher, a newly arriving SU packet will more likely join the system buffer, and the number of SU packets in the system will be greater, so the average latency of the SUs and the blocking ratio of the SUs will increase. If the transmission of this SU packet is interrupted, the interruption ratio of the SUs will be greater.

Figure 6 shows the relationship between the throughput  $S$  of the SUs and the blocking ratio  $P_B$  of the SUs for the different channel access thresholds  $H$ . In this figure, with the blocking ratio growth direction, the channel access probability  $\alpha$  is increasing.

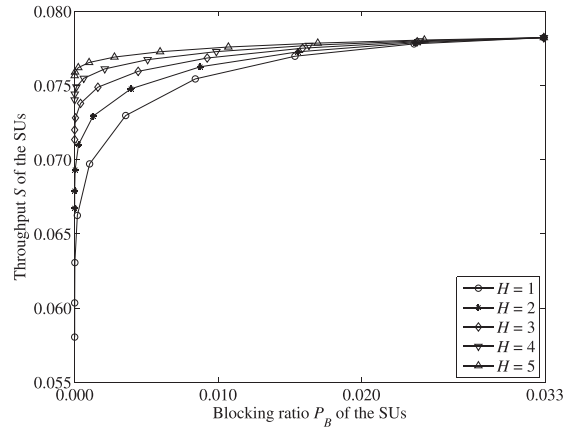


Figure 6: Throughput  $S$  of the SUs vs. blocking ratio  $P_B$  of the SUs ( $\mu_1 = 0.21$ ).

From Fig. 6, we find that for the same channel access threshold  $H$ , as the channel access probability  $\alpha$  increases, the throughput  $S$  of the SUs will increase too. The reason is that the larger the channel access probability is, the more the SU packets joining the system are, and then the greater the throughput of the SUs will be.

On the other hand, for the same channel access probability  $\alpha$ , as the channel access

threshold  $H$  increases, the throughput  $S$  of the SUs will also increase. This is because when the channel access threshold is higher, more SU packets will join the system, and the throughput of the SUs will be higher.

Moreover, we can also observe that as the channel access probability  $\alpha$  increases, the gap between the throughput  $S$  of the SU packets for the different channel access thresholds  $H$  will decrease. The reason is that when the channel access probability is larger, more SU packets can join the system even though the channel threshold is reached, and the influence of the channel access threshold on the throughput will be weaker, so the gap between the throughput for the different channel access thresholds will be smaller.

Figures 7-9 demonstrate how the blocking ratio  $P_B$ , the interruption ratio  $P_I$  and the average latency  $\delta$  of the SUs change with the channel access threshold  $H$  for the different transmission rates  $\mu_1$  of the PU packets, respectively. Moreover, the data set of the channel access threshold  $H$  is supposed to be  $\{1, 2, \dots, 10, 11\}$ . Notably, the case of channel access threshold  $H = 11$  means the channel access threshold is larger than the buffer capacity of the SUs. For this case, all of the SU packets will choose to access the system with probability 1, and the dynamic channel allocation strategy proposed in this paper will be degraded to the conventional channel allocation strategy without channel access probability and channel access threshold. Therefore, the system performance of the dynamic channel allocation strategy and the conventional channel allocation strategy can be compared.

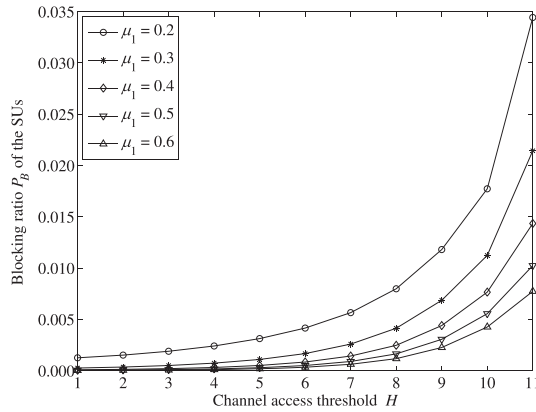


Figure 7: Blocking ratio  $P_B$  of the SUs vs. channel access threshold  $H$  ( $\alpha = 0.5$ ).

In Figs. 7-9, we find that for all the transmission rates  $\mu_1$  of the PU packets, as the channel access threshold  $H$  increases, the blocking ratio  $P_B$ , the interruption ratio  $P_I$  and the average latency  $\delta$  of the SUs will increase. The reasons for these trends have been illustrated in Figs. 3-5.

On the other hand, we notice that for the same channel access threshold  $H$ , along with an increase in the transmission rate  $\mu_1$  of the PU packets, the blocking ratio  $P_B$  and the average latency  $\delta$  of the SUs will decrease, whereas the interruption ratio  $P_I$  of the SUs will increase. This is because for a given channel access threshold, the higher the transmission rate of the PUs is, the faster the PU packets are transmitted, and the higher the possibility is that the SU packets to be transmitted, so the blocking ratio and the average latency of the SUs will decrease. In other words, for a higher transmission rate of the PU packets, the more likely it is that an SU packet will occupy the channel. If this SU packet is interrupted, the interruption ratio of the SUs will be greater.

Additionally, from the cases that the channel access probability is  $\alpha = 1$  in Figs. 3-5, or

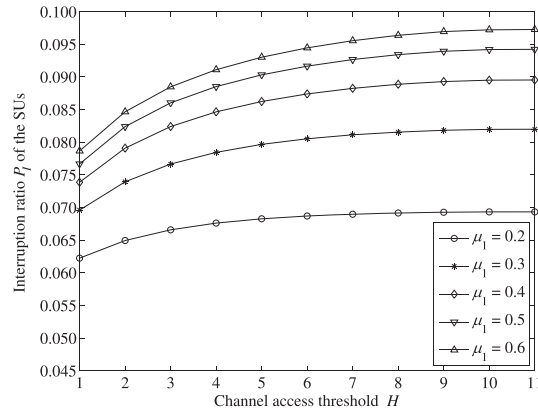


Figure 8: Interruption ratio  $P_I$  of the SUs vs. channel access threshold  $H$  ( $\alpha = 0.5$ ).

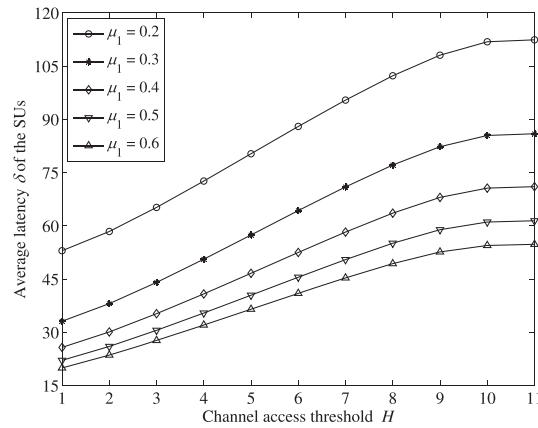


Figure 9: Average latency  $\delta$  of the SUs vs. channel access threshold  $H$  ( $\alpha = 0.5$ ).

the cases that the channel access threshold  $H$  is larger than the buffer capacity  $K = 10$  of the SUs in Figs. 7-9, for example,  $H = 11$ , the system performance of the conventional channel allocation strategy without any channel access probability or channel access threshold can be evaluated. The numerical results show that, in the novel dynamic channel allocation strategy with channel access probability and channel access threshold as proposed in this paper, the blocking ratio, the interruption ratio and the average latency of the SUs can be lowered significantly.

Figure 10 shows the relationship between the throughput  $S$  of the SUs and the blocking ratio  $P_B$  of the SUs for the different transmission rates  $\mu_1$  of the PU packets. In this figure, with the blocking ratio growth direction, the channel access threshold  $H$  is increasing.

From Fig. 10, we find that for the same transmission rate  $\mu_1$  of the PU packets, as the channel access threshold  $H$  increases, the throughput  $S$  of the SUs will increase too. The reason for this trend has been illustrated in Fig. 6.

On the other hand, for the same channel access threshold  $H$ , as the transmission rate  $\mu_1$  of the PU packets increases, the throughput  $S$  of the SUs will also increase. This is because the higher the transmission rate of the PU packets is, the faster the PU packets are transmitted, and the higher the possibility is that the SU packets to be transmitted, so the



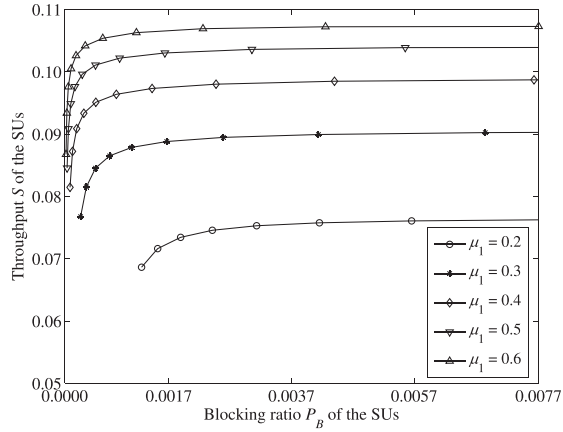


Figure 10: Throughput  $S$  of the SUs vs. blocking ratio  $P_B$  of the SUs ( $\alpha = 0.5$ ).

greater the throughput of the SUs will be.

Moreover, we observe that as the channel access threshold  $H$  increases, the change trend for the throughput  $S$  of the SUs will be more smoothly. The reason for this change trend is that the larger the channel access threshold is, the less likely it is that the number of SU packets in the buffer reaching the channel access threshold, then the relationship between the channel access threshold and the number of SU packets joining the system will be weakened. Therefore, the throughput of the SUs will show a smooth change trend.

In the following part, we show the optimal results by operating the iteration algorithm presented in Table 1. By setting  $C_1 = 300$ ,  $C_2 = 0.01$ ,  $C_3 = 100$ ,  $C_4 = 10$ ,  $\alpha_{min} = 0.01$ ,  $\alpha_{max} = 1$  in the iteration algorithm as an example, the optimal channel access probability  $\alpha^*$  and the corresponding maximum value of  $F(H, \alpha^*)$  for the different channel access thresholds  $H$  and different arrival rates  $\lambda_1$  of the PU packets can be given in Table 2.

Table 2: Optimal channel access probability  $\alpha^*$  and maximum net benefit  $F(H, \alpha^*)$ .

$\lambda_1 = 0.09$	$H$	1	2	3	4	5
	$\alpha^*$	0.93	0.88	0.83	0.75	0.65
	$F(H, \alpha^*)$	26.8088	26.9153	27.0655	27.2549	27.4691
	$H$	6	7	8	<b>9</b>	10
	$\alpha^*$	0.52	0.35	0.15	<b>0.01</b>	0.01
$\lambda_1 = 0.15$	$H$	1	2	3	4	5
	$\alpha^*$	0.70	0.63	0.55	0.45	0.34
	$F(H, \alpha^*)$	21.1257	21.4464	21.7676	22.0531	22.2798
	$H$	6	7	<b>8</b>	9	10
	$\alpha^*$	0.22	0.08	<b>0.01</b>	0.01	0.01
	$F(H, \alpha^*)$	22.4407	22.5401	<b>22.5849</b>	22.5704	22.2958

In Table 2, the estimates of  $\alpha^*$  are accurate to two decimal places, and the estimates of the corresponding  $F(H, \alpha^*)$  are accurate to four decimal places.

From Table 2, we find that when the arrival rate  $\lambda_1$  of the PU packets is 0.09, the optimal solution of  $(H, \alpha)$  is  $(9, 0.01)$ , and the corresponding maximum net benefit  $F(H, \alpha^*)$  is 28.2492. When the arrival rate  $\lambda_1$  of the PU packets is 0.15, the optimal solution of  $(H, \alpha)$

is  $(8, 0.01)$ , and the corresponding maximum net benefit  $F(H, \alpha^*)$  is 22.5849.

Moreover, we see that for the same channel access threshold  $H$ , as the arrival rate  $\lambda_1$  of the PU packets increases, the optimal channel access probability  $\alpha^*$  shows a decrease tendency. Taking  $H = 2$  as an example, for the case of  $\lambda_1 = 0.09$ , we obtain  $\alpha^* = 0.88$ . While for the case of  $\lambda_1 = 0.15$ , we obtain  $\alpha^* = 0.63$ . The reason is that as the arrival rate of the PU packets increases, the blocking ratio, the interruption ratio and the average latency of the SUs will increase accordingly. In order to decrease these performance measures, the channel access probability of the SUs will be set lower.

On the other hand, we also notice that as the arrival rate  $\lambda_1$  of the PU packets increases, the optimal channel access threshold  $H^*$  will decrease. The reason for this trend is the same as the explanation for that of the optimal channel access probability  $\alpha^*$ .

## 7 Conclusions

In this paper, in order to guarantee the QoS of the secondary users (SUs) in cognitive radio networks, we proposed a novel dynamic channel allocation strategy with channel access probability and channel access threshold. We built a discrete-time pre-emptive priority queueing model to capture the working principle of the dynamic channel allocation strategy. By considering the priority of the primary users (PUs) in cognitive radio networks, we analyzed the steady-state distribution of the queueing model with a two-dimensional Markov chain. Accordingly, we derived the formulas for the blocking ratio, the interruption ratio, the throughput and the average latency of the SUs. Considering the different performance measures, we constructed a net benefit function and presented an iterative algorithm to give the optimal designs of the channel access probability and the channel access threshold. The numerical results showed that the dynamic channel allocation strategy with channel access probability and channel access threshold can effectively improve the system performance of the SUs in cognitive radio networks.

The research in this paper provides a theoretical basis for the analysis of dynamic channel allocation strategy, and has potential applications in the performance improvement of cognitive radio networks.

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