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FRACTAL ANALYSIS ON COMPLEX SYSTEM: A CASE STUDY ON GLOBAL STOCK MARKET

CHARUTHA S

Abstract. The study of auto-correlation/cross - correlation and the multifractal behavior of four stock market indices were examined through Multifractal detrended fluctuation analysis (MF - DFA) and Multifractal detrended cross-correlation analysis (MF - DCCA) techniques. Hurst scaling exponents were obtained by MF - DFA and MF - DCCA and observed the persistent nature of the stock indices. The complexity parameters were also calculated from the singularity spectrum obtained from the fluctuation function f(α) vs. holder exponent α plot and examined the complexity nature.

1. Introduction

Complex systems present an interdisciplinary framework that lets in for the invention of the latest ideas, applications, and connections. Benoit Mandelbrot introduced the concept of fractals and coined the term 'Fractals' in 1983 [5]. It has been used in a wide range of applications exhibiting complexity. The two most important characteristics of fractals are self-similarity and non-integer dimension. The long-term correlation in economic time series is particularly decided by the Hurst exponent assessed by different techniques. The prior and most broadly utilized techniques for figuring the Hurst exponent is the Rescaled range analysis method. Hurst proposed this method which is used by Mandelbrot for the time series analysis. Peng et al. put forward a DFA technique to distinguish long-range correlations of the time series. In view of the DFA (Detrended Fluctuation Analysis) techniques, Kantelhardt et al. come up with the Multifractal detrended fluctuation analysis unexpectedly to depict the multifractal highlights of the time series under distinctive time scales.Additionally, the detrended cross-correlation analysis (DCCA) can be eliminated by summarising the elimination trend methodology for double-time series correlation analysis. A method known as DCCA can be used to calculate the long-term correlation of two non-stationary time series. Multifractal detrended cross - correlation analysis (MFDCCA) is a method that Zhou devised for analysing two time series with various fractal patterns. [13].

There are many contemporary research discoveries. Ma et al. provide exact definitions of the cross-correlation between Shanghai and surrounding stock indices, Wang et al. analyse across price returns and trading volumes for the Chinese stock market futures, and Wang and Xie analyse among the Chinese currency and four important currencies. Siokis reports that a multifractal analysis of stock market

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disasters reveals that worldly relationships play a significant effect at a historic occasion. Zebende et al. provide a very clear definition of the relation among the long - range auto - correlation exponents and the long - range cross - correlation exponent; theoretically, this relationship will be strengthened by isolating the DCCA cross-correlation coefficient. Podobnik investigate long-range cross-correlations for a wide range of series, most notably the return series of the New york stock individuals. Numerous academics have examined the multifractal characteristics of various marketplaces. When Matia et al. compared the daily price value of 2449 equities and 29 commodities, they discovered that the multifractal range of price returns for commodities is much larger. Matos et al. employ a distinct approach for analysing the Hurst index with scale and time dependence in order to recover the significant events influencing global markets that may evaluate and take into consideration the practises in newly formed markets. The stock market, the commodities market, and a few other industries were the target of recent probes. Multifractals are employed in a wide range of disciplines in addition to financial markets.

In this paper, BSE, NASDAQ, SHANGHAI, and NIKKEI stock indices is examined and investigated the multifractal behaviour. We look into the cross-correlation and multifractal characteristics of the stock indexes using the MF-DCCA approach. The order of the article follows like, in Section 2, the MF-DCCA approach is briefly introduced, our data is expressed in Section 3. In Section 4, the findings of the analysis are presented, and the analysis is concluded in Section 5.

2. Methodology

In our examination, we utilize the one-dimensional MFDCCA technique to investigate the financial time series data [4, 11, 14]. Assume any two non stationary time series $x(l)$ and $y(l)$ of length M, where $i = 1, 2, ..., M$. The procedure for the one-dimensional MFDCCA method is as follows;

(1) Calculating the cumulative total will enable to determine the time series' profile,

$$X(n) = \sum_{l=1}^{n} [x(l) - \bar{x}]$$
$$Y(n) = \sum_{l=1}^{n} [y(l) - \bar{y}] \tag{2.1}$$

where $\bar{x}$ and $\bar{y}$ are the mean of the time series $x(n)$ and $y(n)$ respectively.

(2) Splitting the time series into $2Ms$ disjoint pieces that are each s in size, with $Ms \equiv M/s$. The trailing short portion of the profile can be deleted because the profile length M is not always a factor of the length of the segment s. To incorporate this section in computation, follow the same procedure that we followed at the end of the profile.

(3) Use a least squares fit of the series to determine the local trend by each fragment. Then calculate covariance

$$F^2_{xy}(p,s) = \frac{1}{s}\sum_{n=1}^{s}\left\{|x[(p-1)s+n]-x_p(n)||y[(p-1)s+n]-y_p(n)|\right\} \tag{2.2}$$

each segment with $p = 1, 2, ...M$ and

$$F^2_{xy}(p,s) = \frac{1}{s}\sum_{n=1}^{s}\left\{|x[M-(p-M_s)s+n]-x_p(n)||y[M-(p-1)s+n]-y_p(n)|\right\} \tag{2.3}$$

each segment with $p = M_s + 1, ...2M$ Here X_p and Y_p are the polynomial trends of the profile in each segment p.

(4) Fluctuation function $F_{xy}(q,s)$ of the detrended cross-correlation with order q and averaged over all the segments, when $q \neq 0$,

$$F_{xy}(q,s) = \left[\frac{1}{2\,M_s}\sum_{p=1}^{2M_s}[F_{xy}(p,s)]^{\frac{q}{2}}\right]^{\frac{1}{q}} \tag{2.4}$$

when $q = 0$

$$F_{xy}(q,s) = exp\left[\frac{1}{4\,M_s}\sum_{p=1}^{2M_s} ln[F_{xy}(p,s)]^{\frac{q}{2}}\right]^{\frac{1}{q}} \tag{2.5}$$

Where q is the moment's order, which is capable of holding any real integer.

(5) The stages II through V are repeated with a different scale size $'s'$ for various estimations of q. The fluctuation function shows the power-law,

$$F_{xy}(q,s) \sim s^{h_{xy}(q)} \tag{2.6}$$

The cross-correlated series exhibits monofractal behaviour when the calculated scaling exponent $h_{xy}(q)$ values are free on q esteems [10, 3]. Additionally, multifractal behaviour appears if the $h_{xy}(q)$ values are depend on the q value. The MFDFA method and the MFDCCA method are identical when the cross-correlated sets of data are about the same, as in when $x = y$. Additionally, the $h_xy(q)$ illustrates the scaling pattern with significant variations for positive q values. While $h_{xy}(q)$ demonstrates the scaling pattern with minor oscillations for negative q values [12, 7, 9, 6]. The $\mathbf{f}_{xy}(\alpha)$ spectrum can also be used to determine whether the data are multifractal. For $\tau_{xy}(q)$, the Legendre transform yields values for $\mathbf{f}_{xy}(\alpha)$:

$$\mathbf{f}_{xy}(\alpha) \equiv q\alpha_{xy} - \tau_{xy}(q) \tag{2.7}$$

Where $\tau_{xy}(q) = qh_{xy}(q)-1$ For monofractal and multifractal series, it is a linear and a nonlinear function, respectively. The multifractal spectral parameters such as the m 卯 ximum position α_0 , the spectrum width $\mathbf{w} = \alpha_{max} - \alpha_{min}$, and the skewness parameter $\mathbf{r} = \frac{(\alpha_{max}-\alpha_0)}{(\alpha_0-\alpha_{min})}$ can be calculated using the multifractal spectrum. For right- and left-skewed spectrums, $\mathbf{r} > 1$ and $\mathbf{r} < 1$, respectively, whereas it is an asymmetric spectrum for $\mathbf{r} = 1$. The right-skewed spectrum is dominated by small fluctuations, while the left-skewed spectrum is dominated by huge fluctuations. [14, 7, 8].

3. Data

The price of BSE, NASDAQ, SHANGHAI, and NIKKEI stock indices are gathered on the daily closing data during a ten-year period from Jan 1, 2010, to Dec 31, 2019. The data has extracted from `https://finance.yahoo.com` and `https://bseindia.com` and has 2123 data points.The logarithmic returns of price were used to find the auto/cross-correlation analysis using MF - DFA and MF - DCCA approaches, respectively [1]. We calculate the logarithmic return in price $s_i(\tau) = log(p_i(\tau + \Delta\tau)) - log(p_i(\tau))$ where $p_i(\tau)$ is the daily price data at time τ, i stands for the time series index and $\Delta\tau = 1$ day.

4. Results and discussion

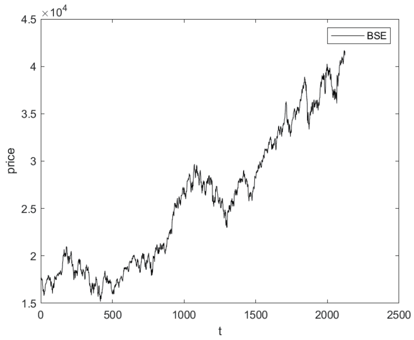
We used MF - DFA and MF - DCCA methodologies to investigate the multifractal behaviour and correlation amoung the stock indices.The logarithmic returns are calculated from the empirical data which is the input series for the MF - DFA and MF - DCCA method. The raw price data and the logarithmic returns of price has shown in the Figure 1.

The scaling exponents $h(q)$ and the fluctuation function $F_q(s)$ were calculated over a range of q values from $q = -10$ to $q = +10$. It is shown in the Figure 2a and Figure 3a. These plots demonstrate the multifractal properties of stock market indices by the high correlation between $h(q)$ and q. Long-range anti-correlated behaviour can be seen in SHANGHAI's scaling exponent. From MF - DCCA, the cross-correlation of BSE - SHANGHAI, NASDAQ - SHANGHAI, SHANGHAI - NIKKEI shows persistent behavior, and BSE - NASDAQ, BSE - NIKKEI, NASDAQ - NIKKEI shows antipersistent behavior. The multifractal scaling exponent $h(q)$ with $q = 2$ are shown in Table 1.

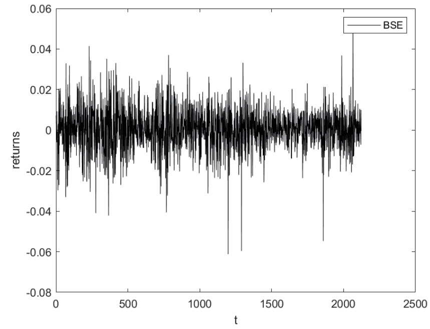
Table 1. The scaling exponent $h(q)$ with $q = 2$ of auto correlated and cross correlated stock indices

	BSE	NASDAQ	SHANGHAI	NIKKIEI
BSE	0.4744			
NASDAQ	0.4486	0.4161		
SHANGHAI	0.5182	0.5101	0.5841	
NIKKIEI	0.4711	0.4674	0.5439	0.4769

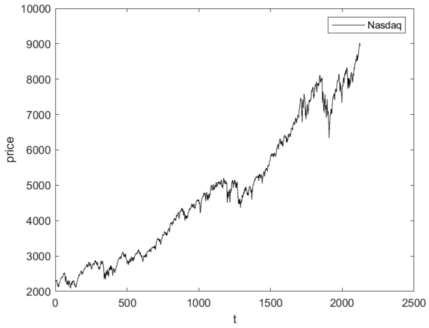
The multifractal spectra of the stock indices is shown in Figure 2b and Figure 3b. It describes the multifractal features of the stock indices. The inverted parabola represents the multifractal series, and for the monofractal time series, it must be



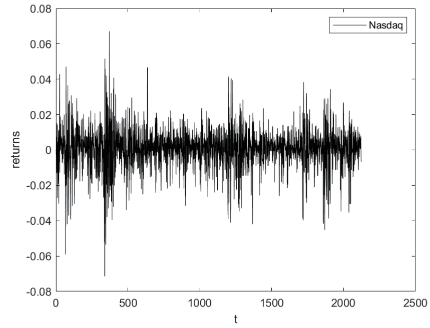
(a) BSE price



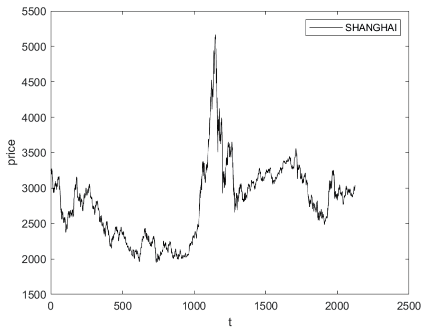
(b) BSE returns



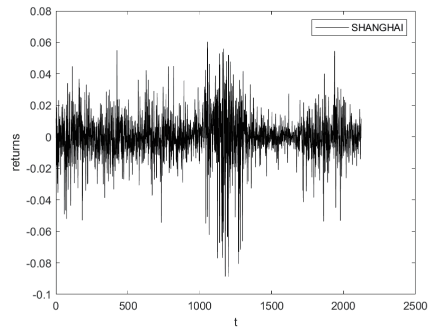
(c) NASDAQ price



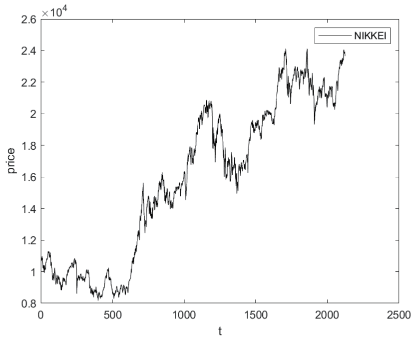
(d) NASDAQ returns



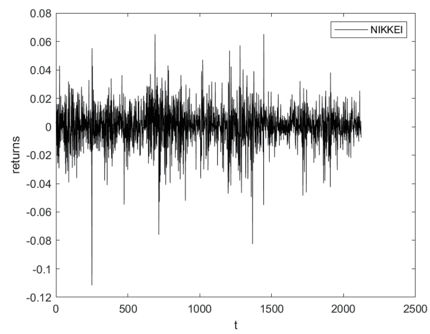
(e) SHANGHAI price



(f) SHANGHAI returns

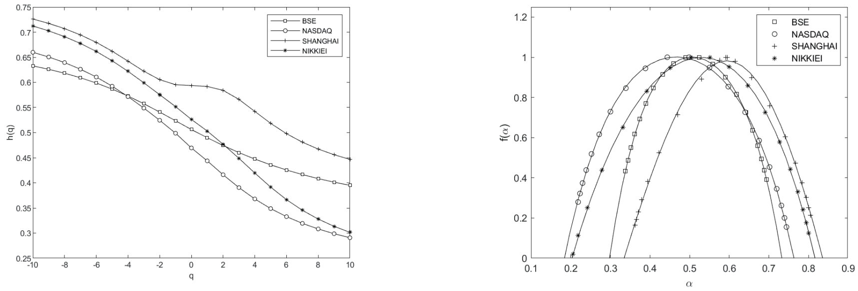


(g) NIKKEI price



(h) NIKKEI returns

FIGURE 1. The raw price data and their returns of the stock indices



(a) Scaling exponents on q for the auto correlated price data

(b) Multifractal spectram of the auto correlated price data

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FIGURE 2. Multifractal analysis of stock indices using MF - DFA

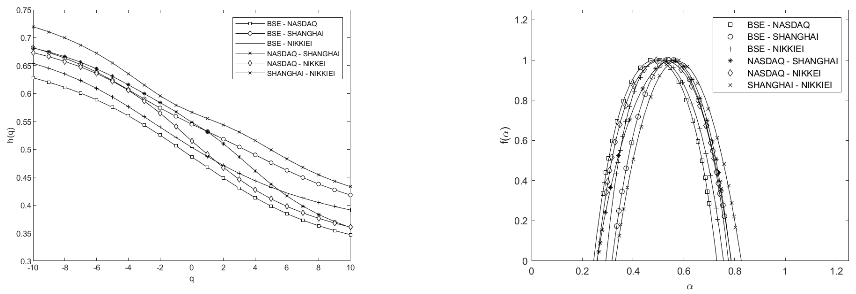

FIGURE 3. Multifractal analysis of stock indices using MF - DCCA

a point. The spectral parameters (α_0, ω, r) can be described from the multifractal spectrum. The multifractal strength of the timeseries, which refers to how complicated the nature of the series is, which is described by the multifractal spectrum width ω. The long-range correlation in the series are represented by the point of the maximum height in the spectrum α_0, and the skew parameter is represented by **r**. The complexity parameters are shown in Table 2. In this research, the complexity parameters(α_0, ω, r) are obtained by fitting the 4^{th} degree polynomial to the multifractal spectrum..

The multifractal spectral parameters are calculated, such as the m 叩 ximum position α_0, the spectrum width $\omega = \alpha_{max} - \alpha_{min}$, and the skew parameter $\mathbf{r} = \frac{(\alpha_{max}-\alpha_0)}{(\alpha_0-\alpha_{min})}$. For the right-skewed spectra $\mathbf{r} > 1$ and for the left-skewed spectra $\mathbf{r} < 1$, whereas for $\mathbf{r} = 1$, it is an asymmetric spectrum.The right-skewed spectra is dominated by small fluctuations, while the left-skewed spectra is dominated by huge fluctuations [2].

The current analysis shows the singularity spectrum of all cross - correlated stocks are a right-skewed, which describes the domination of small fluctuations having maximum position $\alpha_0 \approx 0.57$. The cross-correlated NASDAQ - SHANGHAI spectrum shows broader width compared to others. The spectrum width shows high complexity, i.e., the complexity increases with increasing width.

TABLE 2. The complexity parameters of the cross correlated stock indices

	α_0	ω	r
BSE-NASDAQ	0.4866	0.4856	1.0036
BSE-SHANGHAI	0.5474	0.4706	1.0363
BSE-NIKKEI	0.5007	0.4638	1.2258
NASDAQ-SHANGHAI	0.4879	0.5287	1.2925
NASDAQ-NIKKEI	0.5158	0.5178	1.0093
SHANGHAI-NIKKEI	0.5738	0.4963	1.0359

5. CONCLUSION

The autocorrelation and cross-correlation behavior of BSE, NASDAQ, SHANGHAI, and NIKKEI stock market indices are examined using MF - DFA and MF - DCCA techniques. For both the auto-correlation and cross-correlation of the stock indices, Hurst scaling exponents were determined. SHANGHAI stock market shows persistent behavior, whereas the other three stock market shows anti-persistent behavior for autocorrelation. The cross-correlation of SHANGHAI with the other three stock indices shows persistent behavior, and the rest all show anti-persistent behavior. The complexity parameters were also calculated from the singularity spectrum and found the complex nature of the stock market.

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Charutha S
School of Physics, University of Hyderabad, Gachibowli, Hyderabad 500046, India
E-mail address: `charuthasukumaran93@gmail.com`