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A NOTE ON RELATIONS BETWEEN SKEWNESS AND GEOMETRICAL CONSTANTS OF BANACH SPACES

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Abstract. We study some relations between the skewness $s(X)$ and geometrical constants of Banach spaces X, especially the modulus of smoothness $\rho_X(1)$ and the characteristic of convexity $\varepsilon_0(X)$. A sufficient condition for the inequality $s(X) < 2\rho_X(1)$ is given in terms of $\varepsilon_0(X)$, and an estimate of $s(X)$ from below by $\varepsilon_0(X)$ is also given.

1. Introduction and preliminaries

Let X be a real Banach space with $\dim X \geq 2$, and $S_X = \{x \in X : \|x\| = 1\}$. The *skewness* $s(X)$ of X was introduced by Fitzpatrick and Reznick [3], as follows:

$$s(X) = \sup\Big\{ \lim_{t\to+0} \frac{\|x+ty\| - \|y+tx\|}{t} : \ x, y \in S_X \Big\}.$$

An equivalent definition of $s(X)$ can be found in [3] as the following form:

$$s(X) = \sup\big\{ \langle x, y\rangle - \langle y, x\rangle : \ x, y \in S_X \big\},$$

where $\langle \cdot, \cdot\rangle$ is the "generalized inner product" of Ritt [12]:

$$\langle x, y\rangle = \|x\| \lim_{t\to+0} \frac{\|x+ty\| - \|x\|}{t} \quad (x, y \in X)$$

(cf.[1]). It is obvious that $0 \leq s(X) \leq 2$ for any Banach space X. They showed that X is a Hilbert space if and only if $s(X) = 0$, and X is uniformly non-square if and only if $s(X) < 2$. Moreover, the $s(X)$-constants were calculated for L_p spaces. A modified version of $s(X)$ was investigated by Baronti and Papini [2].

In this paper, we study relations between $s(X)$ and some geometrical constants. We first discuss relations between $s(X)$ and the modulus of smoothness $\rho_X(1)$. It is known that $s(X) \leq 2\rho_X(1)$ for any Banach space X([2]). A sufficient condition for strict inequality in above is given in terms of the characteristic of convexity $\varepsilon_0(X)$. This is an improvement of a result in [9]. Moreover, we give an estimate of $s(X)$ from below by $\varepsilon_0(X)$, which directly gives that $s(X) < 2$ implies $\varepsilon_0(X) < 2$.

We recall some definitions and notations (cf. [4, 7]). A Banach space X is called *uniformly non-square* if there exists $\delta > 0$ such that for any $x, y \in S_X$, either

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$\|x+y\| \le 2(1-\delta)$ or $\|x-y\| \le 2(1-\delta)$. The *James constant* $J(X)$ of X is defined by

$$J(X) = \sup\big\{\min\{\|x+y\|, \|x-y\|\} : x, y \in S_X\big\}.$$

It is obvious that X is uniformly non-square if and only if $J(X) < 2$. The *modulus of convexity* of X is defined by

$$\delta_X(\varepsilon) = \inf\left\{1 - \left\|\frac{x+y}{2}\right\| : x, y \in S_X, \|x-y\| \ge \varepsilon\right\}, \quad 0 \le \varepsilon \le 2.$$

The *characteristic of convexity* of X is defined by

$$\varepsilon_0(X) = \sup\{\varepsilon \in [0,2] : \delta_X(\varepsilon) = 0\}.$$

X is *uniformly convex* if $\delta_X(\varepsilon) > 0$ for all $0 < \varepsilon \le 2$, i.e., $\varepsilon_0(X) = 0$. The *modulus of smoothness* of X is defined by

$$\rho_X(\tau) = \sup\left\{\frac{\|x+\tau y\| + \|x-\tau y\|}{2} - 1 : x, y \in S_X\right\}, \quad \tau \ge 0.$$

The uniform non-squareness is characterized as follows:

Proposition 1.1 ([5])**.** *Let X be a Banach space. The following are equivalent.*
(i) *X is uniformly non-square.*
(ii) $\delta_X(\varepsilon) > 0$ *for some* $0 < \varepsilon < 2$.
(iii) $\varepsilon_0(X) < 2$.
(iv) $\rho_X(1) < 1$.

The following lemmas will be useful later.

Lemma 1.2 ([6])**.** *Let $\{x_n\}, \{y_n\}$ be sequences in a Banach space X such that $\{\|x_n\|\}_{n=1}^{\infty}$ and $\{\|y_n\|\}_{n=1}^{\infty}$ are convergent to non-zero limits, respectively. The following are equivalent.*
(i) $\lim_{n\to\infty} \|x_n + y_n\| = \lim_{n\to\infty}(\|x_n\| + \|y_n\|)$.
(ii) $\lim_{n\to\infty} \left\| \frac{x_n}{\|x_n\|} + \frac{y_n}{\|y_n\|} \right\| = 2$.

Lemma 1.3 ([8])**.** *Let X be a Banach space and $x \in X$ with $x \ne 0$. Then for each y in X, the function*

$$t \mapsto \frac{\|x+ty\| - \|x\|}{t}$$

from $\mathbb{R}\backslash\{0\}$ into $\mathbb{R}$ is non-decreasing.

Lemma 1.4. *Let X be a Banach space. Whenever $\{x_n\}$ and $\{y_n\}$ are sequences in S_X with* $\lim_{n\to\infty}(x_n + y_n) = 0$, *it follows that* $\lim_{n\to\infty} \|x_n - y_n\| = 2$.

Proof. It is clear from the triangle inequality. □

Lemma 1.5. *Let X be a Banach space. Whenever $\{x_n\}$ and $\{y_n\}$ are sequences in S_X with* $\lim_{n\to\infty} \|x_n + y_n\| = 2$, *it follows that* $\overline{\lim}_{n\to\infty} \|x_n - y_n\| \le \varepsilon_0(X)$.

Proof. Since $\{\|x_n - y_n\|\}$ is bounded, we may assume without loss of generality that $\lim_{n\to\infty} \|x_n - y_n\|$ exists. Put $a = \lim_{n\to\infty} \|x_n - y_n\|$. If $a = 2$, then X is not uniformly non-square and hence $a = \varepsilon_0(X) = 2$. Let $a < 2$. Put $u_n = \|x_n - y_n\|$. Since

$$\delta_X(u_n) \le 1 - \left\| \frac{x_n + y_n}{2} \right\|,$$

it follows that $\delta_X(u_n) \to 0$ as $n \to \infty$. Noting that δ_X is continuous on $[0, 2)$ we have $\delta_X(a) = 0$ and so $a \le \varepsilon_0(X)$. This completes the proof. □

2. Results

The following result is due to Baronti and Papini [2].

Proposition 2.1 ([2]). *Let X be a Banach space. Then*

$$s(X) \le 2\rho_X(1). \tag{2.1}$$

If X is not uniformly non-square, then $s(X) = 2\rho_X(1) = 2$. In [9], the first, second authors and Takahashi showed that if X is uniformly convex, then we have strict inequality in (2.1). Note here that there exists some non-uniformly convex (uniformly non-square) space X that we have equality in (2.1). In fact, let X be ℓ_∞-ℓ_1 space, that is, the space $\mathbb{R}^2$ with the norm defined by

$$\|x\| = \begin{cases} \|x\|_\infty & x_1x_2 \ge 0 \\ \|x\|_1 & x_1x_2 \le 0 \end{cases}$$

for $x = (x_1, x_2)$ (cf. [5]). Note that X is not uniformly convex and is uniformly non-square. As in [9], we have $s(X) = 2\rho_X(1) = 1$. The following shows that there exist many non-uniformly convex (uniformly non-square) Banach spaces X that we have strict inequality in (2.1).

Proposition 2.2. *Let X be a Banach space. If $\varepsilon_0(X) \le 1/2$, then $s(X) < 2\rho_X(1)$.*

Proof. Assume that $s(X) = 2\rho_X(1)$. Take sequences $\{x_n\}$ and $\{y_n\}$ in S_X with

$$s(X) - \frac{1}{n} < \langle x_n, y_n \rangle - \langle y_n, x_n \rangle.$$

Let s and t be any numbers in $(0, 1)$. Since $|\langle x, y\rangle| \le \|x\|\|y\|$ for all $x, y \in X$([3]), sequence $\{\langle y_n, x_n\rangle\}$ is bounded and hence we may assume without loss of generality that $\lim_{n\to\infty}\langle y_n, x_n\rangle$ exists. Then, by Lemma 1.3,

$$\begin{aligned}
\langle x_n, y_n \rangle - \langle y_n, x_n \rangle &\le \frac{\|x_n + sy_n\| - \|x_n\|}{s} - \langle y_n, x_n \rangle \\
&\le \frac{\|x_n + y_n\| - \|x_n\|}{1} - \langle y_n, x_n \rangle \\
&\le \|x_n + y_n\| - 1 - \frac{\|y_n - tx_n\| - \|y_n\|}{-t} \\
&\le \|x_n + y_n\| - 1 - \frac{\|y_n - x_n\| - \|y_n\|}{-1}
\end{aligned}$$

$$= \|x_n + y_n\| + \|x_n - y_n\| - 2 \le 2\rho_X(1) = s(X).$$

As $n \to \infty$, it follows that

$$\frac{c-1}{s} = a - 1, \tag{2.2}$$

$$\frac{d-1}{t} = b - 1 \tag{2.3}$$

and

$$s(X) = a + b - 2 = 2\rho_X(1), \tag{2.4}$$

where $a = \lim_{n\to\infty} \|x_n + y_n\|$, $b = \lim_{n\to\infty} \|y_n - x_n\|$, $c = \lim_{n\to\infty} \|x_n + sy_n\|$ and $d = \lim_{n\to\infty} \|y_n - tx_n\|$. If $a = 0$, then $b = 2$ by Lemma 1.4. Hence it follows from (2.4) that $\rho_X(1) = 0$. This is a contradiction to the fact that $\rho_X(1) \ge \sqrt{2} - 1$(cf. [5]). Hence $a > 0$. Similarly, $b > 0$. From (2.2) and (2.3), it follows that

$$\lim_{n\to\infty} \|s(x_n + y_n) + (1 - s)x_n\| = \lim_{n\to\infty} (\|s(x_n + y_n)\| + \|(1 - s)x_n\|) \tag{2.5}$$

and

$$\lim_{n\to\infty} \|t(y_n - x_n) + (1 - t)y_n\| = \lim_{n\to\infty} (\|t(y_n - x_n)\| + \|(1 - t)y_n\|) \tag{2.6}$$

for all s, t in $(0, 1)$. By Lemma 1.2,

$$\lim_{n\to\infty} \left\| \frac{x_n + y_n}{\|x_n + y_n\|} + x_n \right\| = \lim_{n\to\infty} \left\| \frac{y_n - x_n}{\|y_n - x_n\|} + y_n \right\| = 2.$$

As Lemma 1.5 we have

$$\overline{\lim_{n\to\infty}} \left\| \frac{x_n + y_n}{\|x_n + y_n\|} - x_n \right\| \le \varepsilon_0(X) \quad \text{and} \quad \overline{\lim_{n\to\infty}} \left\| \frac{y_n - x_n}{\|y_n - x_n\|} - y_n \right\| \le \varepsilon_0(X).$$

Since $\|x_n + y_n\| \to a$ as $n \to \infty$, we obtain

$$\overline{\lim_{n\to\infty}} \left\| \frac{x_n + y_n}{a} - x_n \right\| \le \varepsilon_0(X) \tag{2.7}$$

by using the triangle inequality. Similarly,

$$\overline{\lim_{n\to\infty}} \left\| \frac{y_n - x_n}{b} - y_n \right\| \le \varepsilon_0(X). \tag{2.8}$$

If $a \le 1$, then it follows from (2.7) that

$$\varepsilon_0(X) \ge \frac{1}{a} \overline{\lim_{n\to\infty}} \|y_n + (1 - a)x_n\| \ge \frac{1}{a}(1 - (1 - a)) = 1.$$

If $b \le 1$, then it follows from (2.8) that

$$\varepsilon_0(X) \ge \frac{1}{b} \overline{\lim_{n\to\infty}} \|x_n - (1 - b)y_n\| \ge \frac{1}{b}(1 - (1 - b)) = 1.$$

Let $a > 1$ and $b > 1$. By (2.3) we have $d > 1$ and hence $\lim_{n\to\infty} \|y_n - tx_n\| > 1$ for all $t \in (0, 1]$. Thus it follows that

$$\varepsilon_0(X) \ge \frac{1}{a} \overline{\lim_{n\to\infty}} \|y_n - (a - 1)x_n\| > \frac{1}{a} \ge \frac{1}{2}$$

by (2.7). This completes the proof. □

As an immediate consequence of this proposition we have the following.

Corollary 2.3 ([9])**.** *Let X be a Banach space. If X is uniformly convex, then $s(X) < 2\rho_X(1)$.*

In [11], Takahashi and Kato gave an estimate $\rho_X(1)$ from above by the James constant $J(X)$.

Proposition 2.4 ([11])**.** *Let X be a Banach space. Then*

$$\rho_X(1) \le 2\Big\{1 - \frac{1}{J(X)}\Big\}.$$

Combining the preceding proposition with Proposition 2.1, we obtain the following.

Proposition 2.5 ([9])**.** *Let X be a Banach space. Then*

$$s(X) \le 4\Big\{1 - \frac{1}{J(X)}\Big\}. \tag{2.9}$$

Combining Proposition 2.2 with Proposition 2.4, we have the strict inequality in (2.9).

Proposition 2.6. *Let X be a Banach space. If $\varepsilon_0(X) \le 1/2$, then*

$$s(X) < 4\Big\{1 - \frac{1}{J(X)}\Big\}.$$

In the following, we consider an estimate the constant $s(X)$ from below by $\varepsilon_0(X)$.

Lemma 2.7. *Let X be a Banach space with $1 \le \varepsilon_0(X) \le 2$ and $0 < t < 1$. Then*

$$s(X) \ge \frac{4t - 2 + \varepsilon_0(X)}{t^2 + t} - 2.$$

Proof. Let $0 < t_0 < 1$. Take sequences $\{u_n\}, \{v_n\}$ in S_X such that $\|u_n - v_n\| \to \varepsilon_0(X)$ and $\|u_n + v_n\| \to 2$. Put $w_n = u_n + t_0 v_n$, $z_n = v_n - t_0 u_n$, $x_n = w_n/\|w_n\|$ and $y_n = z_n/\|z_n\|$ for each n. Since $\|w_n\| \le 1 + t_0$ and $\|w_n\| = \|u_n + v_n - (1 - t_0)v_n\| \ge \|u_n + v_n\| - (1 - t_0)$, we have $\|w_n\| \to 1 + t_0$. Since sequences $\{\|z_n\|\}$, $\{\langle x_n, y_n\rangle\}$, $\{\langle y_n, x_n\rangle\}$ are bounded, we may without loss of generality assume that $\|z_n\| \to a$, $\langle x_n, y_n\rangle \to b$ and $\langle y_n, x_n\rangle \to c$ for some a, b, c. Since $\|z_n\| \le 1 + t_0$ and $\|z_n\| = \|v_n - u_n + (1 - t_0)u_n\| \ge \|u_n - v_n\| - (1 - t_0)$, we have

$$\varepsilon_0(X) - (1 - t_0) \le a \le 1 + t_0 \tag{2.10}$$

and hence it follows from our assumption that $a \ge t_0 > 0$. Also, $\|w_n - t_0 z_n\| = 1 + t_0^2$ and $\|z_n + t_0 w_n\| = 1 + t_0^2$. Thus we have for all t with $0 < t < t_0$,

$$\frac{\|w_n + tz_n\| - \|w_n\|}{t} \ge \frac{\|w_n - t_0 z_n\| - \|w_n\|}{-t_0} = \frac{1 + t_0^2 - \|w_n\|}{-t_0},$$

from which it follows that

$$\langle x_n, y_n\rangle = \frac{1}{\|w_n\|\|z_n\|}\langle w_n, z_n\rangle$$

$$= \frac{1}{\|z_n\|} \lim_{t \to +0} \frac{\|w_n + tz_n\| - \|w_n\|}{t}$$
$$\geq \frac{1}{\|z_n\|} \cdot \frac{1 + t_0^2 - \|w_n\|}{-t_0}.$$

As $n \to \infty$, we have

$$b \geq \frac{1}{a} \cdot \frac{1 + t_0^2 - (1 + t_0)}{-t_0} = \frac{1 - t_0}{a}.$$

From (2.10),

$$b \geq \frac{1 - t_0}{1 + t_0}.$$

On the other hand, for all t with with $0 < t < t_0$,

$$\frac{\|z_n + tw_n\| - \|z_n\|}{t} \leq \frac{\|z_n + t_0 w_n\| - \|z_n\|}{t_0} = \frac{1 + t_0^2 - \|z_n\|}{t_0},$$

from which it follows that

$$\langle y_n, x_n \rangle = \frac{1}{\|z_n\|\|w_n\|} \langle z_n, w_n \rangle = \frac{1}{\|w_n\|} \lim_{t \to +0} \frac{\|z_n + tw_n\| - \|z_n\|}{t}$$
$$\leq \frac{1}{\|w_n\|} \cdot \frac{1 + t_0^2 - \|z_n\|}{t_0}.$$

As $n \to \infty$, we have

$$c \leq \frac{1}{1 + t_0} \cdot \frac{1 + t_0^2 - a}{t_0}.$$

From (2.10),

$$c \leq \frac{1 + t_0^2 - (\varepsilon_0(X) - (1 - t_0))}{(1 + t_0)t_0} = \frac{t_0^2 - t_0 + 2 - \varepsilon_0(X)}{(1 + t_0)t_0}.$$

Thus,

$$\begin{aligned} s(X) &\geq b - c \\ &\geq \frac{1 - t_0}{1 + t_0} - \frac{t_0^2 - t_0 + 2 - \varepsilon_0(X)}{(1 + t_0)t_0} \\ &= \frac{-2t_0^2 + 2t_0 - 2 + \varepsilon_0(X)}{(1 + t_0)t_0} \\ &= \frac{4t_0 - 2 + \varepsilon_0(X)}{t_0^2 + t_0} - 2. \end{aligned}$$

This completes the proof. □

Lemma 2.8. *Let X be a Banach space with $1 \leq \varepsilon_0(X) \leq 2$. We define a function f on $(0, 1)$ as*

$$f(t) = \frac{4t - 2 + \varepsilon_0(X)}{t^2 + t} - 2.$$

Then

$$\sup_{t \in (0,1)} f(t) = 2 + 2(2 - \varepsilon_0(X)) - 2\sqrt{(2 - \varepsilon_0(X))(6 - \varepsilon_0(X))}. \tag{2.11}$$

Proof. Let $\varepsilon_0(X) = 2$. Since f is decreasing on $(0, 1)$, we have

$$\sup_{t\in(0,1)} f(t) = 2$$

and so (2.11). Let $1 \le \varepsilon_0(X) < 2$. Put $d = 2 - \varepsilon_0(X) \in (0, 1]$. Since the derivative of f is

$$f'(t) = \frac{4(t^2 + t) - (4t - d)(2t + 1)}{(t^2 + t)^2} = \frac{-4t^2 + 2dt + d}{(t^2 + t)^2},$$

we have $f'(t_1) = 0$ and f has the maximum at $t = t_1$, where

$$t_1 = \frac{1}{4}(d + \sqrt{d(d + 4)}) \in (0, 1).$$

We now calculate the value $f(t_1)$. By $f'(t_1) = 0$, we have

$$4(t_1^2 + t_1) = (4t_1 - d)(2t_1 + 1),$$

which implies

$$\begin{aligned} f(t_1) &= \frac{4t_1 - d}{t_1^2 + t_1} - 2 = \frac{4}{2t_1 + 1} - 2 = \frac{8}{d + \sqrt{d(d + 4)} + 2} - 2 \\ &= 2\{d + 1 - \sqrt{d(d + 4)}\}. \end{aligned}$$

Thus we obtain (2.11). This completes the proof. □

Remark 2.9. Note that if $\varepsilon_0(X) < 3/2$, then

$$2 + 2(2 - \varepsilon_0(X)) - 2\sqrt{(2 - \varepsilon_0(X))(6 - \varepsilon_0(X))} < 0.$$

Combining Lemma 2.8 with Lemma 2.7, we obtain the main result.

Theorem 2.10. *Let X be a Banach space. Then*

$$s(X) \ge 2 + 2(2 - \varepsilon_0(X)) - 2\sqrt{(2 - \varepsilon_0(X))(6 - \varepsilon_0(X))}. \tag{2.12}$$

Remark 2.11. The inequality (2.12) in the preceding theorem directly gives that $s(X) < 2$ implies $\varepsilon_0(X) < 2$.

Remark 2.12. In [9], the first, second authors and Takahashi estimated $s(X)$ from below by the James constant $J(X)$, as follows:

$$s(X) \ge 2 + 4(2 - J(X)) - 4\sqrt{(2 - J(X))(4 - J(X))}$$

for any Banach space X. We define a function f on $[0, 2]$ as

$$f(t) = 2 + 4(2 - t) - 4\sqrt{(2 - t)(4 - t)}.$$

It is easy to see that f is increasing. Also, Takahashi [10] showed that

$$J(X) \ge \varepsilon_0(X)$$

for any Banach space X. Hence we obtain

$$s(X) \ge f(J(X)) \ge f(\varepsilon_0(X)).$$

Namely,

$$s(X) \ge 2 + 4(2 - \varepsilon_0(X)) - 4\sqrt{(2 - \varepsilon_0(X))(4 - \varepsilon_0(X))}. \tag{2.13}$$

We mention that the inequality (2.12) in Theorem 2.10 is sharper than the inequality (2.13). Indeed, we define a function g on $[0, 2]$ as

$$g(t) = 2 + 2(2 - t) - 2\sqrt{(2 - t)(6 - t)}.$$

It is obvious that $g(t) \geq f(t)$ for all $0 \leq t \leq 2$. Therefore we obtain

$$\begin{aligned} &2 + 2(2 - \varepsilon_0(X)) - 2\sqrt{(2 - \varepsilon_0(X))(6 - \varepsilon_0(X))} \\ &= g(\varepsilon_0(X)) \geq f(\varepsilon_0(X)) \\ &= 2 + 4(2 - \varepsilon_0(X)) - 4\sqrt{(2 - \varepsilon_0(X))(4 - \varepsilon_0(X))}. \end{aligned}$$

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